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Mathematics 30

Information Bulletin

2001–2002 School Year

Diploma Examinations Program

Alberta LEARNING

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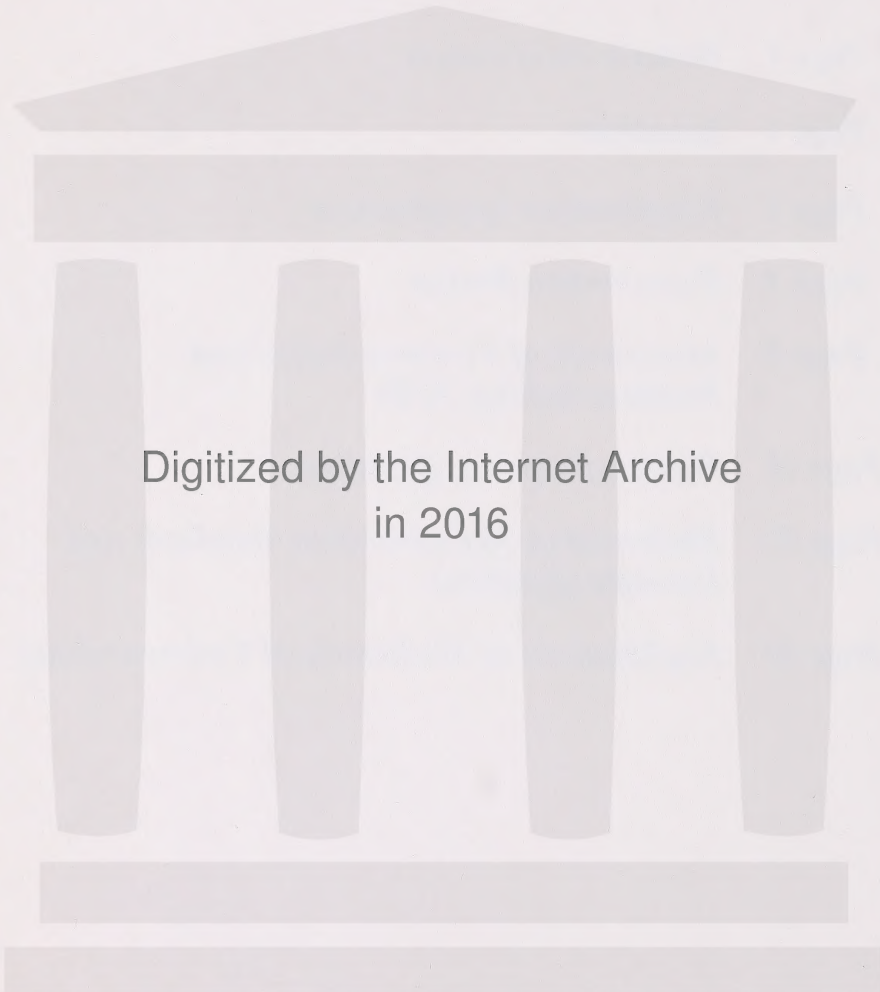
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Bulletin Information

This will be the last bulletin for the Mathematics 30 course, as this course is being phased out. This bulletin applies to all students taking the current Mathematics 30 course in the school year 2001-2002. Diploma examinations for Mathematics 30 will continue during these years.

A separate bulletin is currently available for the Pure Mathematics 30.

Standards

Curriculum Standards

Provincial standards help to communicate how well students need to perform to be judged as having achieved the learnings specified for Mathematics 30. According to the *Mathematics 30 Program of Studies*, student learnings refer to specific knowledge, skill, and attitude expectations. These learnings are amplified on page 25 of this bulletin. These specific statements of standards are written primarily to inform Mathematics 30 teachers about the extent to which students must know the Mathematics 30 content and must demonstrate the required skills to pass the examination. Also included in this bulletin are examples of questions that students should be able to answer to demonstrate the *acceptable standard* or the *standard of excellence*. The examples provided are by no means exhaustive: they are intended to provide a profile of acceptable and excellent achievement.

Achievement Standards

Acceptable Standard

Students who attain the *acceptable standard* but not the standard of excellence in Mathematics 30 will receive a final course mark between and including 50% and 79%. Typically, these students have gained new skills and knowledge in mathematics but can anticipate difficulties if they choose to enroll in post-secondary mathematics courses. They have demonstrated mathematical skills and knowledge in the seven content strands of the Mathematics 30 curriculum and exhibit an ability to apply a broad range of problem-solving skills to these content strands.

Standard of Excellence

Students who attain the *standard of excellence* will receive a final course mark of 80% or higher. Such students have demonstrated their ability and interest in mathematics and feel confident about their mathematical abilities. These students should encounter little difficulty in post-secondary mathematics programs. They should be encouraged to pursue careers in which they will use their talents in mathematics.

Achievement Standards

At a provincial level it is expected that at least 85% of students will achieve a final course mark of 50% or higher. It is also expected that at a provincial level, at least 15% of students will achieve a final course mark of 80% or higher.

Examination Specifications

Each Mathematics 30 Diploma Examination is designed to reflect the core content outlined in the *Mathematics 30 Program of Studies*. The examination is limited to those expectations that can be measured by a paper-and-pencil test. The examination is developed to be completed in 2.5 h; however, students may take an additional 0.5 h to complete the examination.

Students are expected to provide a scientific calculator or a graphing calculator approved by Alberta Learning. Students are also expected to have cleared their calculators of all information that is stored in the programmable or parametric memory.

The content for the Mathematics 30 diploma examinations is emphasized as follows:

Machine-Scored Content[2]	***Percent Emphasis***
Polynomial Functions	11
Trigonometric and Circular Functions	12
Statistics	5
Quadratic Relations	10
Exponential and Logarithmic Functions	11
Permutations and Combinations	10
Sequences and Series	11
Total Multiple-Choice and Numerical-Response Questions	70
Written-Response	30

Written-response questions assess whether or not students can draw on their mathematical experiences to solve problems and explain mathematical concepts. Therefore, the written-response questions will not necessarily fall into a particular unit of study but may cover more than one unit or may require students to make the connections between mathematical concepts.

[2]Content descriptions have been shortened in this table.

The three mathematical understandings of procedures, concepts, and problem solving are addressed throughout the examination. Each understanding has the following approximate emphasis.

Multiple-Choice, Numerical-Response, and Written-Response	*Percent Emphasis*
Procedures	34
Concepts	33
Problem Solving	33

Since the machine-scored and written-response questions are weighted components, calculate a student's examination mark using the following equation:

$$\left(\frac{x}{49} \times 70\right) + \left(\frac{y}{15} \times 30\right) = \text{exam mark}$$

x = Student total score on multiple-choice and numerical-response questions
y = Student total score on written-response questions

Examination Design

The design of the Mathematics 30 diploma examinations is as follows:

Question Format	*Number of Questions*	*Percent Emphasis*
Multiple Choice	40	57
Numerical Response	9	13
Written Response	3	30

The examinations consist of a multiple-choice, numerical-response section, and a written-response section.

The numerical-response questions are interspersed throughout the multiple-choice questions, according to content topic.

Information required to answer multiple-choice or numerical-response questions is often located in a box preceding the question. On occasion, the information given in the box may be required to answer more than one question. The number of questions that require the use of the information given in the box will be clearly stated above the box; e.g., "*Use the following information to answer the next two questions.*"

Multiple Choice and Numerical Response

For multiple-choice questions, students are to choose the correct or best possible answer from four alternatives.

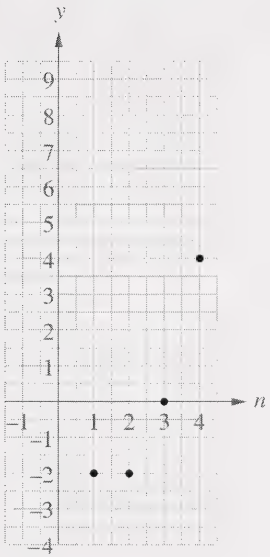
For numerical-response questions, students are to calculate a numerical answer. As well, students are to record their answer in a separate area of the answer sheet. When the answer to be recorded cannot be a decimal value (e.g., the number of people, the degree of a polynomial), students are asked to determine a whole number value (e.g., the number of people is _____, the degree of this polynomial is _____.) If the answer can be a decimal value, then students are asked to record their answer correct to the nearest tenth or nearest hundredth.

Numerical-response questions may not require students to actually calculate a numerical answer from a problem, but may ask students to record their understanding of a conceptual idea. The following is an example of such a question.

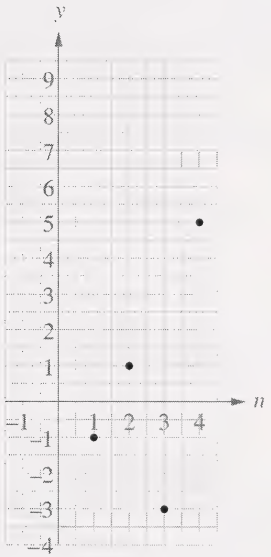
Use the following information to answer the next question.

Four graphs, numbered 1 to 4 inclusive, are shown below.
Each point $P(a, b)$ satisfies the condition $a \in N$, $b \in \mathrm{I}$.

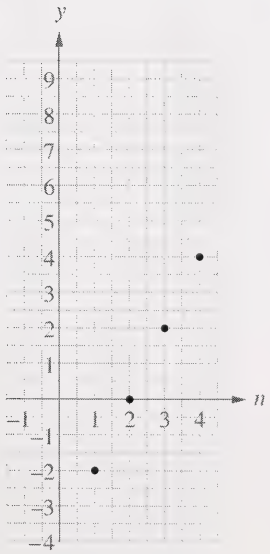
Graph 1



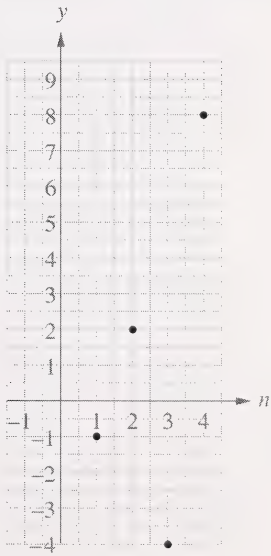
Graph 2



Graph 3



Graph 4



Numerical Response

2. Each of the statements below describes one of the graphs numbered 1 to 4. Match each statement with the graph it describes.

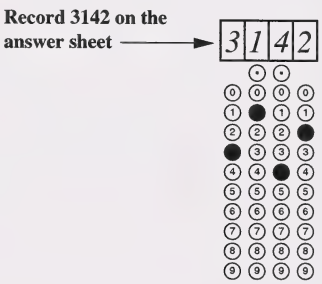
Statement A
Each point on the graph is in the form (n, t_n), where t_n, $1 \le n \le 4$, is an **arithmetic sequence**. _____ (Record in column 1 on the answer sheet)

Statement B
Each point on the graph is in the form (n, S_n), where S_n is the **sum** of the **arithmetic sequence** in statement ***A***. _____ (Record in column 2 on the answer sheet)

Statement C
Each point on the graph is in the form (n, t_n), where t_n, $1 \le n \le 4$, forms a **geometric sequence**. _____ (Record in column 3 on the answer sheet)

Statement D
Each point on the graph is in the form (n, S_n), where S_n is the **sum** of the **geometric sequence** in statement ***C***. _____ (Record in column 4 on the answer sheet)

Answer: 3142



Written Response

The written-response section focuses on students' understanding of the process of solving a problem and encourages students to take risks to arrive at a solution. Students will be rewarded for selecting a problem-solving strategy and for carrying through with the strategy to find a solution. To achieve the *standard of excellence*, students must be able to select a strategy, carry it through, and solve the problem correctly. The written-response section of the examination also focuses on students' understanding of mathematical concepts and allows for flexibility in evaluating students' communication and problem-solving abilities in mathematics.

Questions in the written-response section ask students to solve, explain, justify, or prove their solution.

In scoring the written-response section of the examinations, markers will evaluate how well students

- understand the problem or the mathematical concept
- correctly use the mathematics
- use problem-solving strategies and explain their answer and procedures
- communicate their solutions and mathematical ideas

Above all, students should be encouraged to try to solve the problem. Even an attempt at a solution could be worth some marks. If students leave the paper blank, markers will not be able to award any marks. The three written-response questions are each scored with a five-mark scoring rubric. Samples of student responses and corresponding justifications for questions from the January 1998 examination will be presented beginning on page 23.

Assessment of Communication and Problem-Solving Skills

The open-ended question provides a way in which to examine mathematics as communication and mathematics as problem solving. An open-ended question allows students to communicate a response by asking them to explain their reasoning, explain their solution, describe mathematical situations, write directions, create new problems, create new strategies, generalize a mathematical situation, and formulate hypotheses.

Communication

In keeping with the expectations listed in the *Mathematics 30 Program of Studies*, the examinations will reflect mathematics as communication. The program of studies includes communication in the problem-solving expectations: "Students will be expected . . . to read the problem thoroughly; identify and clarify key components; restate the problem, using familiar terms. . . ask relevant questions; . . . document the solution process . . . and explain the solution in oral or written form . . . ." (From *Mathematics 30 Program of Studies*, pages 6–7)

These expectations are consistent with the recommendations made by the National Council of Teachers of Mathematics:

> In grades 9 to 12, the mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can
>
> - reflect upon and clarify their thinking about mathematical ideas and relationships
> - formulate mathematical definitions and express generalizations discovered through investigations
> - express mathematical ideas orally and in writing
> - read written presentations of mathematics with understanding
> - ask clarifying and extending questions related to mathematics they have read or heard about
> - appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas
>
> Focus: All students need extensive experience listening to, reading about, writing about, speaking about, reflecting on, and demonstrating mathematical ideas.
>
> (From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, page 140)

The National Council of Teachers of Mathematics describes the evaluation of mathematics as communication in the following manner:

> The assessment of students' ability to communicate mathematics should provide evidence that they can

- express mathematical ideas by speaking, writing, demonstrating, and depicting them visually
- understand, interpret, and evaluate mathematical ideas that are presented in written, oral, or visual forms
- use mathematical vocabulary, notation, and structure to represent ideas, describe relationships, and model situations

(From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, page 214)

Besides writing to communicate results, the accuracy of and logic in students' mathematical statements also reflects mathematics as communication. Hence, students will continue to be expected to demonstrate logical and meaningful communication on the diploma examinations.

Problem Solving

In keeping with the expectations identified in the *Mathematics 30 Course of Studies*, the examinations will reflect mathematics as problem solving. Problem solving is integrated throughout the content areas in the curriculum. A set of specific problem-solving learner expectations precedes the specific content learner expectations.

The expectations contained in the program of studies are consistent with the recommendations made by the National Council of Teachers of Mathematics:

In grades 9 to 12, the mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can

- use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content
- apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics
- recognize and formulate problems from situations within and outside mathematics
- apply the process of mathematical modeling to real-world problem situations

Focus: In grades 9 to 12, the problem-solving strategies learned in earlier grades should have become increasingly internalized and integrated to form a broad basis for the student's approach to doing mathematics, regardless of the topic at hand. From this perspective, problem solving is much more than applying specific techniques to the solution of classes of word problems. It is a process by which the fabric of mathematics as identified in later standards is both constructed and reinforced.

(From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, page. 137)

Alberta Learning recommends the following two documents, which examine the open-ended question in further detail.

Assessment Alternatives in Mathematics: An overview of assessment techniques that promote learning

A Question of Thinking: A First Look at Students' Performance on Open-ended Questions in Mathematics

Both documents are available from

California State Department of Education
Bureau of Publications, Sales Unit
P.O. Box 271
Sacramento, CA 95802-0271

The following document was recently published by the National Council of Teachers of Mathematics. It provides some practical suggestions for the assessment of problem solving and communication on a regular basis.

Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions

This document is available from

National Council of Teachers of Mathematics
1906 Association Drive
Reston, VA 22091-1593

Written-Response Questions

This section of the bulletin provide a general scoring guide, sample written-response questions, question-specific scoring guides, and sample student responses and rationales. The two questions are from the January 1998 diploma examination.

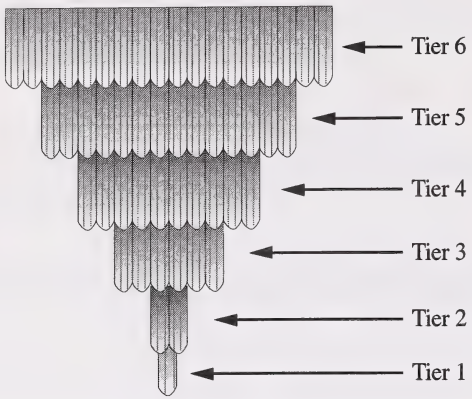
Scoring Guide for Written-Response Questions

Credit may be given to students who show unusual insight. If their solutions fall outside *specific question scoring rubrics*, they will be scored against the *General Scoring Guide* shown below.

GENERAL SCORING GUIDE	
5 marks	The student • demonstrates an understanding of the problem • uses appropriate mathematical knowledge and problem-solving techniques to find the solution • justifies the solution and explains the relevance to the problem
4 marks	The student • demonstrates an understanding of the problem • uses appropriate mathematical knowledge and problem-solving techniques to find a solution but the solution contains a minor flaw • shows some justification of his or her results
3 marks	The student • demonstrates some understanding of the problem • uses appropriate mathematical knowledge and problem-solving techniques to find partial solutions • communicates little understanding of the complexities of the problem • formulates some aspects of the problem mathematically
2 marks	The student • explores the initial stages of the problem • applies some relevant mathematical knowledge and problem-solving techniques to find partial solutions
1 mark	The student • explores the initial stages of the problem • applies some relevant mathematical knowledge and problem-solving techniques in working toward solutions

Use the following information to answer the next question.

A lighting store specializes in custom-made chandeliers built in tiers. Each tier consists of a number of rectangular glass pieces arranged in a circular formation.



The chandelier shown is not the exact one described in this written-response question.

Written Response — 10%

2. The numbers of pieces of glass in each successive tier of a chandelier form a geometric sequence. Every chandelier uses 1 glass piece for tier 1. In a chandelier with 5 or more tiers, there are 81 glass pieces in tier 5.

- Complete the chart below by indicating the number of glass pieces required for tiers 2, 3, and 4, and by indicating the total number of glass pieces required for a chandelier consisting of 2, 3, 4, or 5 tiers.

Tier Number (n)	**Number of glass pieces in n^{th} tier**	**Total number of glass pieces in a chandelier with n tiers**
1	1	1
2		
3		
4		
5	81	

- The cost for each glass piece is $1.75. Determine the cost of the glass pieces required for a 7-tiered chandelier.

- Write a general expression of the total cost of the glass pieces required for a chandelier of n tiers, where $n \in N$.

- The manufacturer of the glass pieces has determined that the width of each piece is normally distributed about a mean of 4.00 cm with a standard deviation of 0.05 cm. Any piece with a width less than 3.90 cm or more than 4.10 cm cannot be used for a chandelier. If 100 000 glass pieces are selected at random for chandeliers, how many pieces from this initial selection will not meet the size requirement?

Question 2—Scoring Guide	
5 marks	The student • correctly completes the problem
4 marks	The student finds a solution that contains a minor flaw such as • an incorrect number in the chart **OR** • an arithmetical error in the cost of a 7-tier chandelier **OR** • rejecting $1 \times 100\,000(0.0228)$ pieces of glass
3 marks	The student obtains a partial solution by • correctly completing the first three bullets **OR** • correctly completing any two of the first three bullets and making significant progress on the fourth bullet **OR** • correctly completing the fourth bullet and at least one of bullets 1, 2, or 3, and making significant progress on two of bullets 1, 2, or 3 **OR** • correctly completing the fourth bullet and consistently using a geometric sequence or arithmetic series throughout the first three bullets
2 marks	The student explores the initial stages to find a partial solution by • correctly completing one of bullets 2, 3, or 4 **OR** • correctly completing the chart and making significant progress on at least one other bullet **OR** • making significant progress on two of bullets 2, 3, or 4 **OR** • consistently and correctly using a geometric sequence or arithmetic series throughout the first 3 bullets (4^{th} bullet not done) **OR** • correctly completing the chart, using t_n in bullets 2 and 3, and making significant progress on the last bullet
1 marks	The student only explores the initial stages of the problem; i.e., some progress is made on one of bullets 2, 3, or 4 **OR** • the student correctly completing the chart

Note: For bullet 4, "significant progress" means going beyond the probability associated with the z-score.

Question 1

Example 1

- **Complete the chart below by indicating the number of glass pieces required for tiers 2, 3, and 4, and by indicating the total number of glass pieces required for a chandelier consisting of 2, 3, 4, or 5 tiers.**

Tier Number (n)	Number of glass pieces in n^{th} tier	Total number of glass pieces in a chandelier with n tier
1	1	1
2	3	4
3	9	13
4	27	40
5	81	121

- **The cost for each glass piece is \$1.75. Determine the cost of the glass pieces required for a 7-tiered chandelier.**

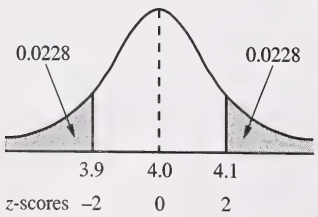
$$S_n = \frac{a(r^n - 1)}{r - 1} \qquad 1\,093 \times 1.75 = 1\,912.75$$

$$S_7 = \frac{(1)(3^7 - 1)}{3 - 1} = 1093 \qquad \text{Cost is } \$1\,912.75$$

- **Write a general expression of the total cost of the glass pieces required for a chandelier of n tiers, where $n \in N$.**

$$C(n) = 1.75 \times \left(\frac{3^n - 1}{2}\right) = 0.875(3^n - 1)$$

- **The manufacturer of the glass pieces has determined that the width of each piece is normally distributed about a mean of 4.00 cm with a standard deviation of 0.05 cm. Any piece with a width less than 3.90 cm or more than 4.10 cm cannot be used for a chandelier. If 100 000 glass pieces are selected at random for chandeliers, how many pieces from this initial selection will not meet the size requirement?**



$$z = \frac{x - \mu}{\sigma} \qquad z = \frac{4.1 - 4}{0.05} = 2 \qquad z = \frac{3.9 - 4}{0.05} = -2$$

The rejected number of glass pieces is $100\,000 \times 2 \times 0.0228 = 4\,560$.

Examples 1 and 2 score – 5

Rationale

The students correctly completed the chart, have calculated the cost of the glass pieces required for a 7-tiered chandelier, wrote a general expression for the total cost of the glass pieces required for a chandelier with n tiers, and used the given statistical information correctly to determine the number of glass pieces that would not meet the size requirement.

Example 2

The numbers of pieces of glass in each successive tier of a chandelier form a **geometric sequence**. Every chandelier uses 1 glass piece for tier 1. In a chandelier with 5 or more tiers, there are 81 glass pieces in tier 5.

- Complete the chart below by indicating the number of glass pieces required for tiers 2, 3, and 4, and by indicating the total number of glass pieces required for a chandelier consisting of 2, 3, 4, or 5 tiers.

Tier Number (n)	Number of glass pieces in n^{th} tier	Total number of glass pieces in a chandelier with n tiers
1	1	1
2	3	4
3	9	13
4	27	40
5	81	121
6	243	364
7	729	1093

Continued

Question 1

- The cost for each glass piece is $1.75. Determine the cost of the glass pieces required for a 7-tiered chandelier.

As shown in an extension of the chart given on the previous page, there are 729 glass pieces in the 7th tier. In a 7-tier chandelier there are a total of 1093 pieces.

Thus, total cost = $1093 \text{ pieces} \times \frac{\$1.75}{\text{piece}}$

$= \$1912.75$

- Write a general expression of the total cost of the glass pieces required for a chandelier of n tiers, where $n \in N$.

The # of glass pieces in a chandelier of n tiers is the sum of a geometric series where $a = 1$ and $r = 3$. Because the total cost of the chandelier # of glass pieces $\times$ unit cost:

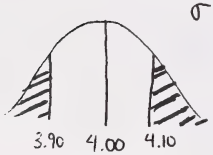
$$\text{total cost} = 1.75[S_n] = 1.75\left[\frac{a(r^n-1)}{r-1}\right] = 1.75\left[\frac{1(3^n-1)}{2}\right] = 1.75\left[\frac{(3^n-1)}{2}\right]$$

where $n \in N$ and total cost is given in dollars

- The manufacturer of the glass pieces has determined that the width of each piece is normally distributed about a mean of 4.00 cm with a standard deviation of 0.05 cm. Any piece with a width less than 3.90 cm or more than 4.10 cm cannot be used for a chandelier. If 100 000 glass pieces are selected at random for chandeliers, how many pieces from this initial selection will not meet the size requirement?

$z = \frac{x-\mu}{\sigma}$

$\sigma = 0.05$



$$z_{4.10} = \frac{4.10 - 4.00}{0.05} = 2.00$$

$$z_{3.90} = \frac{3.90 - 4.00}{0.05} = -2.00$$

area under blue $= 0.5000 - 0.4772$
$= 0.0228$

area under red $= 0.5000 - 0.4772$
$= 0.0228$

total area shaded $= 0.0456$ $\therefore$ P(rejection) $= 0.0456$

~~Written-response question 3 begins on the next page.~~

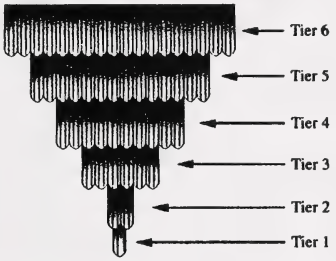
rejected = P(rejection) $\cdot$ # in sample

$= 0.0456 \times 100\,000$

$= 4560$

Example 3

A lighting store specializes in custom-made chandeliers built in tiers. Each tier consists of a number of rectangular glass pieces arranged in a circular formation.



The chandelier shown is not the exact one described in this written-response question.

Written Response — 10%

2. The numbers of pieces of glass in each successive tier of a chandelier form a geometric sequence. Every chandelier uses 1 glass piece for tier 1. In a chandelier with 5 or more tiers, there are 81 glass pieces in tier 5.

- Complete the chart below by indicating the number of glass pieces required for tiers 2, 3, and 4, and by indicating the total number of glass pieces required for a chandelier consisting of 2, 3, 4, or 5 tiers.

Tier Number (n)	Number of glass pieces in n^{th} tier	Total number of glass pieces in a chandelier with n tiers
1	1	1
2	21	22
3	41	63
4	61	124
5	81	205

Continued

Score – 3

Rationale
The student correctly completed the fourth bullet and consistently used an arithmetic series throughout the first three bullets.

Question 1

- The cost for each glass piece is $1.75. Determine the cost of the glass pieces required for a 7-tiered chandelier.

427 × 1.75

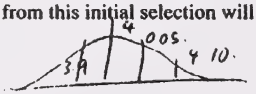
$747.25

- Write a general expression of the total cost of the glass pieces required for a chandelier of n tiers, where $n \in N$.

$(10n - 9)n$

$1.75(10n^2 - 9n) = T$ (Total cost)

- The manufacturer of the glass pieces has determined that the width of each piece is normally distributed about a mean of 4.00 cm with a standard deviation of 0.05 cm. Any piece with a width less than 3.90 cm or more than 4.10 cm cannot be used for a chandelier. If 100 000 glass pieces are selected at random for chandeliers, how many pieces from this initial selection will not meet the size requirement?



$-2 = \frac{3.9 - 4}{0.05}$

$2 = \frac{4.1 - 4}{0.05}$

$0.4772 \times 2 = 0.9544$

$1 - 0.9544 = 0.0456$

$100,000 \times 0.0456 = 4560$ pieces

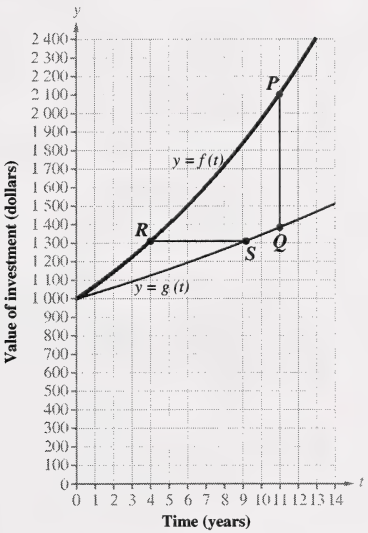
Use the following information to answer the next question.

Student A and student B each deposited \$1 000 in different investment plans.

The partial graph of the exponential function f approximates the value of the \$1 000 that student A deposited into a plan earning 7%/a interest compounded annually.

The partial graph of the exponential function g approximates the value of the \$1 000 that student B deposited into a plan earning i%/a interest compounded annually.

Both graphs are shown below.



Written Response — 10%

3. The function $f(t) = 1\,000(1.07)^t$, $t \in N$, represents the value in dollars, after t years, of the \$1 000 that student A invested at 7%/a compounded annually.

- Determine algebraically the minimum amount of time it will take for student A's investment to be worth at least \$2 100.

  Explain how the graph supports or does not support your answer.

- The point (2, 1060.90) lies on the graph of $y = g(t)$. Algebraically determine the interest rate.

- By referring to the **two** investments described in the problem, explain what the length of the vertical line segment $\overline{PQ}$ and the length of the horizontal line segment $\overline{RS}$ represent.

Question 3—Scoring Guide	
5 marks	The student shows a complete solution by • algebraically determining how long it will take for the money to be worth \$2 100 and indicates how the graph supports the answer **AND** • algebraically determining the interest rate is 3% **AND** • giving a clear explanation of the significance of line segments $\overline{PQ}$ and $\overline{RS}$
4 marks	The student finds a solution but it contains a minor flaw; for example, the student • does not state how the graph supports the answer **OR** • does not write the solution as 3% **OR** • does not realize $t \in N$ in first bullet **OR** • has made a computational error in either of the first 2 bullets
3 marks	The student finds a partial solution by • completing 2 bullets correctly **OR** • completing 1 bullet correctly and making a significant start on the other 2
2 marks	The student explores the initial stages to find partial solution by • completing 1 bullet correctly **OR** • making significant progress on at least 2 of the bullets
1 marks	The student explores only the initial stages by • applying some relevant mathematical knowledge to one of the bullets

Note: If a student doesn't indicate time as fixed for $\overline{PQ}$ **and** the value of investments being equal for $\overline{RS}$ in the explanation for bullet 3, the student has **not** answered the 3rd bullet.

Question 2

Example 1

- **Determine algebraically the minimum amount of time it will take for student A's investment to be worth at least $2 100.**

  Explain how the graph supports or does not support your answer.

$$F(t) = 1\,000(1.07)^t$$
$$2\,100 = 1\,000(1.07)^t$$
$$\frac{\log 2.1}{\log 1.07} = t$$

It will take 11 years.

The graph of $y = f(t)$ appears to pass through (11, 2 100); therefore, it supports my answer.

- **The point (2, 1060.90) lies on the graph of $y = g(t)$. Algebraically determine the interest rate.**

$$g(t) = 1\,000\left(1+\frac{i}{100}\right)^t,$$

where I = the interest rate expressed as a percent

$$1\,060.90 = 1\,000\left(1+\frac{i}{100}\right)^2$$
$$1.0609 = \left(1+\frac{i}{100}\right)^2$$
$$3 = i$$
$$y = 14.65\cos\left(\frac{\pi}{6}t\right)+1.15$$

- **By referring to the two investments described in the problem, explain what the length of the vertical line segment $\overline{PQ}$ and the length of the horizontal line segment $\overline{RS}$ represent.**

The length of line segment $\overline{PQ}$ represents the difference in the amounts of the two investments at a particular point in time.

The length of line segment $\overline{RS}$ represents the difference in time required for the two investments to reach the same value.

Examples 1 and 2 score – 5

Rationale

The students determined algebraically the minimum amount of time it would take for student A's investment to be worth at least $2 100, explained how the graph supported the answer, algebraically determined the interest rate, and clearly explained what the length of $\overline{PQ}$ and $\overline{RS}$ represented.

Example 2

The function $f(t) = 1\,000(1.07)^t$, $t \in N$, represents the value in dollars, after t years, of the \$1 000 that student A invested at 7%/a compounded annually.

- Determine algebraically the minimum amount of time it will take for student A's investment to be worth at least \$2 100.

Explain how the graph supports or does not support your answer.

Given that $f(t) = 1000\,(1.07)^t$, we want to determine what value of t yields \$2100. Therefore:

$$\frac{2100}{1000} = \frac{1000}{1000}(1.07)^t \qquad \text{solving for } t$$

$$2.1 = 1.07^t$$

$$\log 2.1 = t \log 1.07$$

$$t = \frac{\log 2.1}{\log 1.07}$$

$$= 10.965\ldots$$

Therefore, it takes about 11 years for the investment to be worth \$2100.

This is supported by the graph. At $t = 11$ years, conveniently shown by point P, the student's investment $f(t)$ is shown to be worth about \$2100.

Continued

Question 2

$x = 2$
$y = 1\,060.90$

- The point (2, 1 060.90) lies on the graph of $y = g(t)$.
  Algebraically determine the interest rate.

$y = g(t)$

because $g(t)$ is also an interest rate compounded annually

$g(t) = \text{initial investment (rate)}^{t}$ (initial investment also is 1000)

$1\,060.90 = 1000\,(r)^{2}$

$1.0609 = r^{2}$

$r = \pm 1.03$ but interest rates are not negative (hopefully)

$\therefore r = 1.03$, or 3% /a

- By referring to the **two** investments described in the problem, explain what the length of the vertical line segment $\overline{PQ}$ and the length of the horizontal line segment $\overline{RS}$ represent.

Vertical line segment $\overline{PQ}$ represents the difference in value of investment for student A and student B at $t = 11$, or after 11 years. The longer the line segment is, the more difference there is between the value of investment A and the value of investment B. Referring to the graph, $\overline{PQ}$ is 700 units long. Therefore, because the y axis is scaled in dollars, investment A is $700 more valuable than B.

Horizontal segment $\overline{RS}$ represents length of time it takes investment B to reach the value of investment A. It takes A 9 years to reach $1300, but investment B ≈ 9 years to reach the same amou
The length of $\overline{RS}$ is the difference in time. Because $\overline{RS}$ is 5 units, ∴ 5 years, it takes investment B 5 years longer to reach $1300.

Example 3

3. The function $f(t) = 1\,000(1.07)^t$, $t \in N$, represents the value in dollars, after t years, of the \$1 000 that student A invested at 7%/a compounded annually.

- Determine algebraically the minimum amount of time it will take for student A's investment to be worth at least \$2 100.

Explain how the graph supports or does not support your answer.

$$2100 = 1000(1.07)^t$$
$$\frac{2100}{1000} = 1.07^t$$
$$2.1 = 1.07^t$$
$$\log 2.1 = t \log 1.07$$
$$\frac{\log 2.1}{\log 1.07} = t$$
$$10.9 = t$$

The graph supports the answer. As t represents time, we take a look at where 10.9 yrs lies and find out where student A's investment graph line crosses 10.9 yrs and find that it corresponds with \$2100 just as the equation showed us.

- The point (2, 1 060.90) lies on the graph of $y = g(t)$. Algebraically determine the interest rate.

$$1060.90 = 1000(i)^2$$
$$\frac{1060.90}{1000} = i^2$$
$$1.0609 = i^2$$

- By referring to the **two** investments described in the problem, explain what the length of the vertical line segment $\overline{PQ}$ and the length of the horizontal line segment $\overline{RS}$ represent.

$\overline{PQ}$ shows how much more student A has made over student B in the period of 11 years. $\overline{RS}$ shows how much more quickly student A's investment will reach \$1300 as opposed to student B's investment.

Score – 3

Rationale
There is significant progress on the first two bullets even though the student did not recognize that $t \in N$ and neglected to round the value of time to 11 years. The student completed the third bullet correctly but did not solve for i in the second bullet.

Mathematics 30 Curriculum Standards and Example Questions

Mathematics 30 Curriculum Standards

The Curriculum Standards provided in this appendix are intended to clarify the Mathematics 30 Course of Studies *statements. Included are some examples of questions that students must be able to do to demonstrate* the acceptable standard *or* the standard of excellence. *For a definition of these* standards, *see page 1 of this bulletin.*

***Note: For the complete documentation of Mathematics 30 Learner Expectations, please refer to the* Mathematics 30 Course of Studies.**

Problem Solving

Students in Mathematics 30 can participate in and contribute toward the problem-solving process for problems within the seven content strands.[1]

Examples

- Given the solution to a problem, analyze the solution for correctness, provide the correct response, and provide possible reasons for errors. For example:

  Menghsha examined the graph of the function $y = 3 \sin \theta$ and determined that the domain of the function was $-1 \leq \theta \leq 1$. Is Menghsha's answer correct? If not, provide the correct answer and explain Menghsha's error.

- Given one method of solving a problem, solve it a second way. For example:

  Jillian was asked to find the factors of $P(x) = x^3 - 9x^2 - x + 9$. On her graphing calculator, she graphed the function and determined that the factors for $P(x)$ were $(x - 9)$, $(x + 1)$, and $(x - 1)$. If Jillian was unable to graph $P(x)$, show another method that Jillian could have used to find its factors.

- Given a problem, solve it for the specific case(s) and then provide a general solution. For example:

  Given a 4-sided and a 10-sided polygon, determine the number of diagonals in each. Also determine a general statement about the number of diagonals in an n-sided polygon.

[1] Italicized comments give an overview of the curriculum statements to be found in the *Mathematics 30 Course of Studies.*

Polynomial Functions

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• recognize and give examples of polynomial functions of different degrees	
• generate the graph of any integral polynomial function with the use of graphing calculators or graphing utility packages	
• use the Remainder Theorem to evaluate a third-degree integral polynomial function for rational values of the variable, and understand how this can be used to find factors of the polynomial function	• use the Remainder Theorem when either the factor or the original polynomial contains unknown coefficients • use the Remainder Theorem to evaluate an integral polynomial function beyond the third degree for rational values of the variable, and understand how this can be used to find factors of the polynomial function
• factor and find the zeros for an integral polynomial function in standard form, degree 3 or less, in which all zeros are rational	
• by using graphing calculators or computers, find approximations for all the real zeros of integral polynomial functions	
• derive an equation of an integral third-degree polynomial function, given its rational zeros	• derive an equation for an integral polynomial function, given its zeros and any other information that will uniquely define it
• recognize the general shape of graphs of integral polynomial functions of degree four or less where the multiplicity of zeros is one, two, or three	• recognize the general shape of graphs of integral polynomial functions of degree n where the multiplicity of zeros is greater than two • explain the relationships between the graphs of different polynomial functions and their zeros

- identify the potential rational zeros of an integral polynomial function
- determine the minimum degree of a polynomial function by using the multiplicities of its zeros
- participate in and contribute toward the problem-solving process for problems that can be represented by polynomial functions studied in Mathematics 30
- complete the solution to problems that can be represented by polynomial functions studied in Mathematics 30

Examples

- Given $P(x) = 10x^3 + 51x^2 + 3x - 10$, determine its zeros, its factors, and its graph. The student can also describe the relationship among its zeros, its factors, and its graph.
- Given $P(x) = ax^3 + bx^2 + 3$ and that the remainders are 7 and 10 when divided by $x - 2$ and $x + 1$ respectively, determine the zeros, the factors, and the graph of $P(x)$.

Trigonometric and Circular Functions

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• convert angle measurements between degree and radian measure	
• given any two of the following measurements—the radian measure of the central angle, the radius, or the length of an arc—determine the unknown measurement	
• verify[2] the fundamental trigonometric identities	• prove[3] trigonometric identities
• solve first-degree trigonometric equations on the domain $0 \leq \theta < 2\pi$ in radians and $0° \leq \theta < 360°$	• solve first- and second-degree trigonometric equations, including double and half angles on the domain $0 \leq \theta < 2\pi$ and $0° \leq \theta < 360°$
• simplify and evaluate simple trigonometric expressions involving the fundamental trigonometric identities	
• generate the graph of trigonometric functions with the use of graphing calculators or graphing utility packages	
• explain the effect of each parameter a, b, c, and d on the graph of the $y = a \sin[b(\theta + c)] + d$ and $y = a \cos[b(\theta + c)] + d$ functions	• explain, orally and in writing, the combined effects of the parameters a, b, c, and d in the trigonometric functions $y = a \sin[b(\theta + c)] + d$ and $y = a \cos[b(\theta + c)] + d$, on the functions' domain and range
• state the domain and range of $y = \sin\theta$, $y = \cos\theta$, and $y = \tan\theta$	

[2]For a definition of **verify**, see the *Mathematics 30 Course of Studies*, p. 6.

[3]For a definition of **prove,** see the *Mathematics 30 Course of Studies*, p. 6.

- describe, orally and in writing, the relationship between the root(s) of a first-degree trigonometric equation and the graph of its corresponding function
- participate in and contribute toward the problem-solving process for problems that can be represented by trigonometric functions studied in Mathematics 30

- describe, orally and in writing, the relationship between the root(s) of a trigonometric equation and the graph of its corresponding function
- complete the solution to problems that can be represented by trigonometric functions studied in Mathematics 30

Examples

- Given $y = 2\sin\left(\theta - \frac{1}{2}\right), 0 \le \theta < 2\pi$, determine its zeros, describe orally and in writing, the relationship between its zeros and its corresponding graph. So, describe the effect that 2 and $-\frac{1}{2}$ have on the graph of $y = \sin\theta$.
- Given $\frac{\cot\theta}{\tan\theta}$, use fundamental trigonometric identities to simplify the expression. Verify this simplification by substituting values for the variable and by comparing their corresponding graphs.

- Given $2 - 2\cos^2\theta = \sin\theta,\ 0 \le \theta < 2\pi$, determine its zeros and describe, orally and in writing, the relationship between its zeros and the graphs of $y = 2 - 2\cos^2\theta$ and $y = \sin\theta$.

Statistics

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• describe the characteristics of normally distributed data	
• interpret the mean and standard deviation of a set of normally distributed data	
• apply the standard normal curve and the z-scores of data that are normally distributed	
• participate in and contribute toward the problem-solving process for problems that require the analysis of statistics studied in Mathematics 30	• complete the solution to problems that require the analysis of statistics studied in Mathematics 30

Examples

• Given that the results of a test were normally distributed with a mean of 30 and a standard deviation of 5, and given that the passing mark was set at 25, determine the percentage of students who passed the test.	• Given that the results of a test were normally distributed with a mean of 30 and a standard deviation of 5, and given that the passing mark was set at 25, determine the percentage of students who passed the test.

Quadratic Relations

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• describe, orally, in writing, and by modelling, the intersection of a plane and a conical surface that would result in a hyperbola, an ellipse, a parabola, and a circle	
• describe orally, in writing, and by modelling, and identify the position of the plane at which the intersection of a plane and a conical surface defines a degenerate ellipse and hyperbola	• describe orally, in writing, and by modelling, and identify the position of the plane at which the intersection of a plane and a conical surface defines a degenerate parabola
• generate the graph of quadratic relations with the use of graphing calculators or graphing utility packages	
• describe, orally and in writing, and identify the quadratic relation defined by a combination of numerical coefficients for any quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$	
• describe, orally and in writing, and identify the quadratic relation formed when given the value of the eccentricity	
• describe, orally and in writing, and identify the eccentricity when given the quadratic relation	• describe, orally and in writing, the combination of values for the numerical coefficients of the general quadratic relation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, that would result in the degenerate conics

- describe, orally and in writing, and identify the quadratic relation formed when given the locus definition
- identify and graph the quadratic relation when given a point on the quadratic relation, a fixed point, and the eccentricity
- calculate the eccentricity when given a fixed horizontal or vertical line, a fixed point, and a point on the quadratic relation
- identify and graph the quadratic relation when given the eccentricity, a fixed point, and a fixed horizontal or vertical line
- describe, orally and in writing, and identify the effects on the graph of the quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when two of the numerical coefficients change
- participate in and contribute toward the problem-solving process for problems that require the analysis of quadratic relations studied in Mathematics 30

- describe, orally and in writing, and identify the changes in the graph of a quadratic relation when the eccentricity changes
- use the locus definition to verify the equation of each conic section
- describe, orally and in writing, and identify the effects on the graph of the quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when two or more of the numerical coefficients change
- complete the solution to problems that require the analysis of quadratic relations studied in Mathematics 30

Examples

- Given $2x^2 + 2y^2 + x - 3y - 25 = 0$, identify the quadratic relation described by this equation.

- Given $Ax^2 + Cy^2 + Dx - Ey - 36 = 0$, identify this as a hyperbola when $AC < 0$.

- Given that a quadratic relation is represented by $3x^2 + 4y^2 + 5x + Ey - 36 = 0$, where $B = 0$, describe, orally and in writing, what happens to the graph of this quadratic relation when 5 is changed to –4 and –36 is changed to –9.

- Given a quadratic relation that is described as having an eccentricity of 2, identify this as a hyperbola and describe its locus.

- Given a locus of points such that the sum of the distances between one of the points and two fixed points is constant, identify this locus as an ellipse.

- Given a description of the intersection of a plane and a conical surface, identify the conic section formed.

- Given that the cutting plane approaches the vertex of the conical surface, describe, orally, in writing, and by modelling, the effect on the ellipse.

- Given that the eccentricity of the orbit of Halley's comet, which has a period of 76 years, is 0.96, sketch its graph.

- Describe and identify the effects on the graph of the quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when two or more of the numerical coefficients change.

- Given a description of the intersection of a plane and a conical surface, identify orally and in writing, the degenerate parabola formed.

- Given the equation of a degenerate quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, describe, orally and in writing, the quadratic relation formed.

- Given the graph and the eccentricity of a quadratic relation, describe, orally and in writing, the changes to the graph when the eccentricity changes.

- Given that the fixed point of an ellipse is moving closer to the centre of the ellipse, describe, orally and in writing, the effect on the eccentricity.

- Given that the eccentricity of any quadratic relation is the ratio of the distance between any point and a fixed point to the distance between the same point and a fixed line, describe, orally and in writing, the effect that changing the eccentricity has on the relative positions of the fixed line and the fixed point.

Exponential and Logarithmic Functions

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• generate the graph of exponential and logarithmic functions with the use of graphing calculators or graphing utility packages	
• recognize and sketch the graphs of exponential and logarithmic functions and recognize their inverse relationship	
• convert functions from exponential form to logarithmic form and vice versa	
• apply the laws and properties of logarithms to evaluate logarithmic expressions	
• solve and verify simple exponential and logarithmic equations	• solve and verify exponential and logarithmic equations
• state the domain and range of the exponential and logarithmic functions	
• use the graphs of the exponential and logarithmic functions to estimate the value of one of the variables, given the other variable	
• participate in and contribute toward the problem-solving process for problems that can be represented by logarithmic or exponential functions studied in Mathematics 30	• complete the solution to problems that can be represented by logarithmic or exponential functions studied in Mathematics 30

Examples

- Given $f(x) = 4^{2x}$, sketch its graph, describe its domain and range, find the zeros of its corresponding equation, and describe, orally and in writing, the relationship between the roots of its equation and its graph. Write the inverse of $f(x) = 4^{2x}$ in logarithmic form, sketch its graph, describe its domain and range, determine the zeros of this equation, and describe, orally and in writing, the relationship between the zeros of its equation and its graph.

- Given the equation $\log_5(x - 4) + \log_5(x - 2) = 3$, find all the possible values of x, identify the domain, describe, orally and in writing, the relationship between the roots of this equation and its graph, and describe, orally and in writing, the reasons why there are values of x that satisfy the equation but that are not permissible for the function.

Permutations and Combinations

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• calculate the number of linear, circle, and ring permutations and permutations with repetitions of n things taken r at a time	• explain the reason that there are different numbers of permutations when a given number of objects are arranged in a line, a circle, or a ring, or when some of the objects are repeated or identical
• calculate the number of combinations of n things taken r at a time	
• expand binomials of the form $(x + a)^n$, $n \in W$, using the binomial theorem	• expand binomials of the form $(x + by)^n$, $n \in W$, using the binomial theorem, and determine specific terms of this expansion
• describe, orally and in writing, the difference between a permutation and a combination	
• participate in and contribute toward the problem-solving process for problems involving permutations and/or combinations, including probability problems, studied in Mathematics 30	• complete the solution to problems involving permutations and/or combinations, including probability problems, studied in Mathematics 30

Examples

- Given that there are 10 musicians in the finals of a music competition, decide whether permutations or combinations should be used to calculate the number of ways in which first, second, and third place prizes can be awarded.

- Given the binomial $(x + 2)^5$, find the coefficient of the x^3 term in the expansion, determine how the coefficient of the term containing $x^4(2)$ is obtained, determine the number of terms in the expansion of $(x + 2)^5$, and describe, orally and in writing, the relationship between the number of terms in the expansion and the exponent of the binomial.

- Given that five people can sit at a round table, decide whether permutations or combinations should be used to determine the number of different orders in which these five can sit at the table, if Jack and Jill must sit next to one another. Justify the method of solution.

- Given the binomial $(3x - 2y)^7$, find the coefficient of the x^3 term in the expansion, determine how the coefficient of the term containing $[(3x)^6(2y)]$ is obtained, and determine the number of terms in the expansion of $(3x - 2y)^7$. Describe, orally and in writing, the relationship between the number of terms in the expansion and the exponent of the binomial, how the number of a given term in the expansion of $(3x - 2y)^7$ relates to the exponent of $(2y)$ in that term, and how the coefficients of the terms that are equidistant from the ends of the expansion of $(3x - 2y)^7$ compare in terms of combinations.

Sequences and Series

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• write the specific terms of a sequence, given its defining function	• write the specific terms of a sequence when given its recursive definition • determine the functions describing any sequence that has a recognizable pattern
• expand a series given in sigma notation	
• describe, orally and in writing, the difference between sequences and series, arithmetic or geometric, infinite or finite	
• apply the general term "formula" to arithmetic and geometric sequences	• solve problems using the general term and/or sum formulas in which there are two unknowns
• apply the sum formula for arithmetic and geometric series	
• participate in and contribute toward the problem-solving process for problems involving sum and term formulas for arithmetic and geometric series and sequences studied in Mathematics 30	• complete the solution to problems involving sum and term formulas for arithmetic and geometric series and sequences studied in Mathematics 30

Examples

- Given that a sequence is defined by $t_n = 5n - 3$, write the terms of the sequence, determine whether the sequence is arithmetic or geometric, determine the common difference or common ratio, determine the related series, and determine the sum of a specified number of terms.

- Given that a series is defined by $\sum_{n=3}^{6}(-2)^n$, write the terms of the series, determine whether the series is arithmetic or geometric, and determine the sum of the series.

- Given that a sequence is defined by $t_n = 5n - 3$, write the terms of the sequence, determine whether the sequence is arithmetic or geometric, determine the common difference or common ratio, determine the related series, determine the sum of a specified number of terms, and determine the formula for the sum of n terms.

- Given an arithmetic sequence where $t_4 + t_{13} = 99$ and $t_7 = 39$, determine the first term of this sequence.

Explanation of Mathematical Understandings

Procedures

The assessment of students' knowledge of *mathematical procedures* should provide evidence that they can:

- recognize when a procedure is appropriate
- give reasons for the steps in a procedure
- reliably and efficiently execute procedures
- verify the results of procedures empirically (e.g., using models) or analytically
- recognize correct and incorrect procedures
- generate new procedures and extend or modify familiar ones
- appreciate the nature and role of procedures in mathematics

It is important that students know how to execute mathematical procedures reliably and efficiently: a knowledge of procedures involves much more than simple execution. Students must know when to apply them, why they work, and how to verify that they have given a correct answer; they also must understand concepts underlying a procedure and the logic that justifies it. Procedural knowledge also involves the ability to differentiate those procedures that work from those that do not, and the ability to modify them or create new ones. Students must be encouraged to appreciate the nature and role of procedures in mathematics; that is, they should appreciate that procedures are created or generated as tools to meet specific needs in an efficient manner and thus can be extended or modified to fit new situations. The assessment of students' procedural knowledge, therefore, should not be limited to an evaluation of their facility in performing procedures: it should emphasize all the aspects of procedural knowledge addressed in this standard.

Concepts

The assessment of students' knowledge and understanding of *mathematical concepts* should provide evidence that they can:

- label, verbalize, and define concepts
- identify and generate examples and non-examples
- use models, diagrams, and symbols to represent concepts
- translate from one mode of representation to another
- recognize the various meanings and interpretations of concepts
- identify properties of a given concept and recognize conditions that determine a particular concept
- compare and contrast concepts

In addition, assessment should provide evidence of the extent to which students have integrated their knowledge of various concepts.

An understanding of mathematical concepts involves more than mere recall of definitions and recognition of common examples: it encompasses the broad range of abilities identified in this standard. Assessment, too, must address these aspects of conceptual understanding. Assessment tasks should focus on students' abilities to discriminate between the relevant and the irrelevant attributes of a concept in selecting examples and non-examples, and to represent concepts in various ways. Tasks that ask students to apply information about a given concept in novel situations provide strong evidence of students' knowledge and understanding of that concept. Problems designed to elicit information about students' misconceptions can provide information useful in planning or modifying instruction.

Problem Solving

The assessment of students' ability to use mathematics in *solving problems* should provide evidence that they can:

- formulate problems
- apply a variety of strategies to solve problems
- solve problems
- verify and interpret results
- generalize solutions

Students' ability to solve problems develops over time as a result of extended instruction, opportunities to solve many kinds of problems, and encounters with real-world situations. Students' progress should be assessed systematically, deliberately, and continually to effectively influence their confidence and ability to solve problems in various contexts. Assessments should determine students' ability to perform all aspects of problem solving. Evidence about their ability to ask questions, use given information, and make conjectures is essential to determine if they can formulate problems. Assessments should also yield evidence of students' use of strategies and problem-solving techniques and of their ability to verify and interpret results. Finally, because the power of mathematics is derived, in part, from its generalizability, this aspect of problem solving should be assessed as well.

From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, p. 209, p. 223, p. 228.

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